Section 6.6: Logarithmic and exponential equations

#1 - 12: Solve the exponential equation by writing each side of the equation with the same base then equating the exponents. Problems also may be solved with logarithms.

$16 = 2^4$

1) $2^x = 16$

$2^x = 2^4$

$\boxed{x = 4}$

$32 = 2^5$

3) $2^{x+1} = 32$

$2^{x+1} = 2^5$

$x + 1 = 5$

$-1 \quad -1$

$\boxed{x = 4}$

5) $\left(\frac{1}{2}\right)^{x} = 16$

$(2^{-1})^{x} = 2^{4}$

$2^{-1x} = 2^{4}$

$\frac{-1x}{-1} = \frac{4}{-1}$

$x = -4$

7) $2^{4-x} = 64$

$2^{4-x} = 2^{6}$

$4 - 1x = 6$

$-4 \quad -4$

$\frac{-1x}{-1} = \frac{2}{-1}$

$x = -2$

9) $32^x = 2$

$(2^5)^x = 2^1$

$2^{5x} = 2^1$

$\frac{5x}{5} = \frac{1}{5}$

$x = \frac{1}{5}$

11) $16^x = 4$

$(2^4)^x = 2^2$

$2^{4x} = 2^2$

$\frac{4x}{4} = \frac{2}{4}$

$x = \frac{1}{2}$

#13 - 24: Solve the exponential equations, round your answer to 2 decimals.

Steps:

1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)

2) Write in logarithmic form

3) Check

13) $3^x = 6$

$\log_3 6 = x$

Steps:

1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)

2) Write in logarithmic form (replace log_e $with$ ln

3) Check

15) $e^x = 12$

$\log_e 12 = X$

Rewrite $\log_e = Ln$

$Ln(12) = X$

Steps:

1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)

2) Write in logarithmic form

3) Check

17) $5(10^x) = 20$

$\frac{5}{5} \quad \frac{5}{5}$

$10^x = 4$

$\log_{10} 4 = x$

Rewrite $\log_{10} = \log$

$\boxed{x = \log 4}$

Steps:

1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)

2) Write in logarithmic form

3) Check

19) $32e^{2x} = 128$

$\frac{32}{32} \quad \frac{32}{32}$

$e^{2x} = 4$

$\log_e 4 = 2x$

$\frac{\ln(4)}{2} = \frac{2x}{2}$

$\boxed{x = \frac{\ln(4)}{2}}$

21) $3^{x-1} = 5$

$\log_3 5 = x - 1$

$+1 \qquad +1$

Steps:

1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)

2) Write in logarithmic form

3) Check

$1 + \log_3 5 = x$

Steps:

1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)

2) Write in logarithmic form

3) Check

23) $6^{x+4} = 9$

$\log_6 9 = x+4$

$-4 \qquad -4$

$-4 + \log_6 9 = x$

#25 - 51: Solve the logarithmic equations, round to 2 decimals when needed.

Steps (#25 – 38)

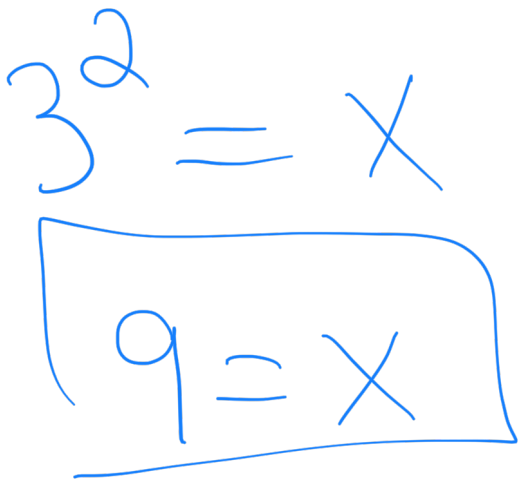
1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

25) $\log_3 x = 2$



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- place the argument to the right of the equal sign.

2) Solve for x

3) Check

27) ln x = 1

$\log_e x = 1$

$e^1 = x$

$\boxed{x = e}$

Steps (#25 – 38)

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

29) $\log_x 49 = 2$ $(x > 0)$

$\sqrt{x^2} = \pm\sqrt{49}$

$x = \pm 7$

only choose $x = 7$ base must be positive

$\boxed{x = 7}$

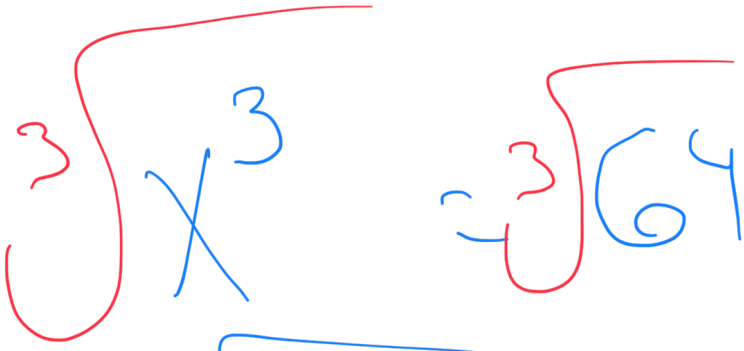
Steps (#25 – 38)

1) Rewrite the problem in exponential form.

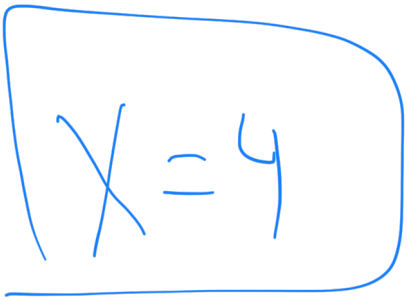
- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check



31) $\log_x 64 = 3$ $(x > 0)$



Steps (#25 – 38)

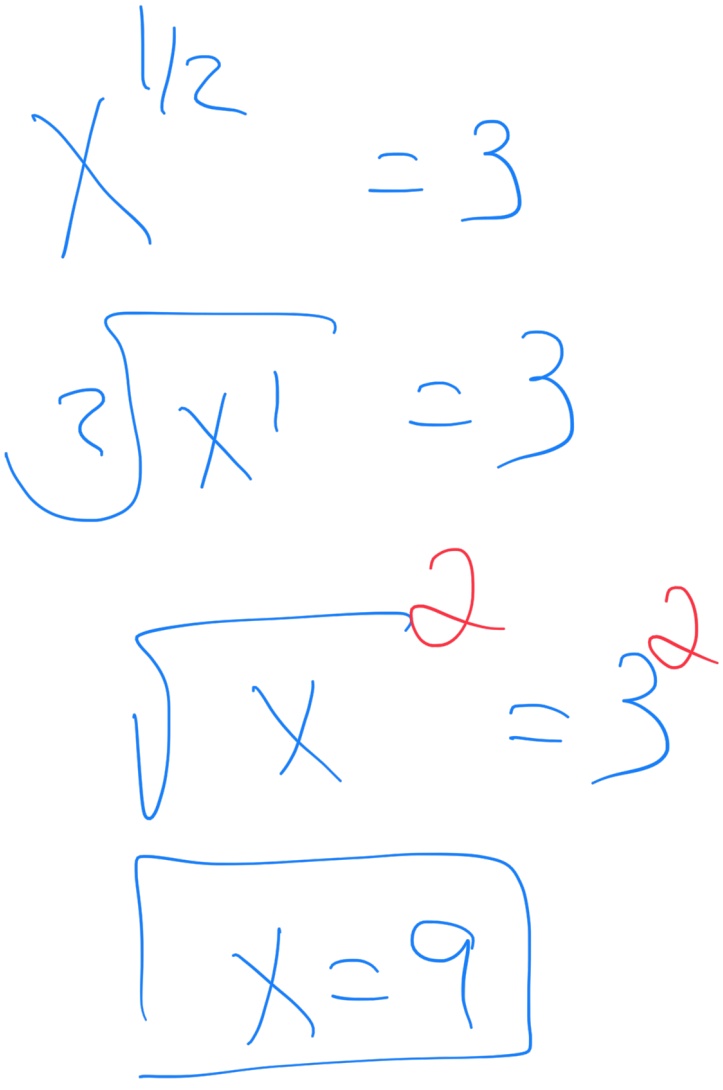
1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

33) $\log_x 3 = \frac{1}{2}$ (x>0)



Steps (#25 – 38)

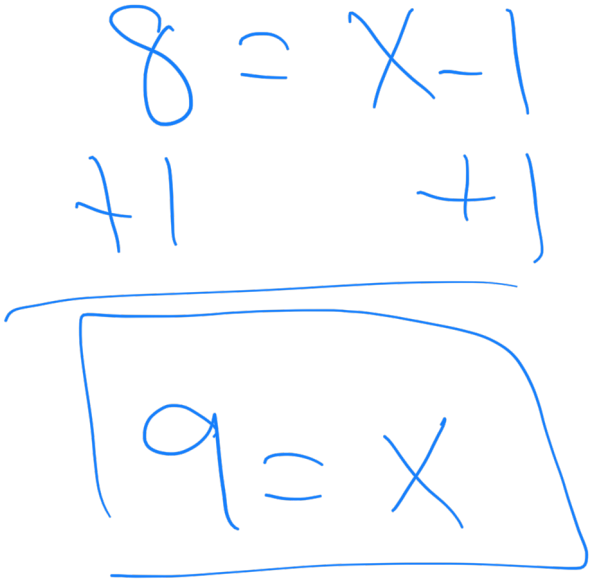
1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

35) $\log_2 (x-1) = 3$



Steps (#25 – 38)

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

$2^5 = 2x$

37) $\log_2(2x)=5$

$\frac{32}{2} = \frac{2x}{2}$

$16 = x$

Steps: (#39 – 42)

1) Drop the logs and set the arguments equal to each other.

2) Solve for x.

3) Check.

39) log (x+1) = log (3x–2)

$$x+1 = 3x-2$$
$$-x \qquad -x$$
$$1 = 2x-2$$
$$+2 \qquad +2$$
$$3 = 2x$$
$$\frac{3}{2} = \frac{2x}{2}$$
$$\boxed{\frac{3}{2} = x}$$

Steps: (#39 – 42)

1) Drop the logs and set the arguments equal to each other.

2) Solve for x.

3) Check.

41) $\log_2(x+3) = \log_2(3x)$

$$x+3 = 3x$$
$$-x \qquad -x$$
$$3 = 2x$$
$$\frac{3}{3} = \frac{2x}{3}$$
$$\boxed{3/2 = x}$$

Steps: (#43 – 47)

1) Use the minus to divide rule to write the left side with one logarithm.

2) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base to of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

3) Solve for x (you will need to cross multiply to solve)

4) Check

43) $\log_2 x - \log_2 (x+6) = -2$

$$\log_2 \frac{x}{x+6} = -2$$

$$2^{-2} = \frac{x}{x+6}$$

$$\frac{1}{2^2} = \frac{x}{x+6}$$

$$\frac{1}{4} = \frac{x}{x+6}$$

$$x+6 = 4x$$

$$-x \quad -x$$

$$6 = 3x$$

$$2 = x$$

Steps: (#43 – 47)

1) Use the minus to divide rule to write the left side with one logarithm.

2) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

3) Solve for x (you will need to cross multiply to solve)

4) Check

$$\log_2 \frac{x}{x-6} = 2$$

45) $\log_2 x - \log_2 (x - 6) = 2$

$$2^2 = \frac{x}{x-6}$$

$$4 = \frac{x}{x-6}$$

$$\frac{4}{1} = \frac{x}{x-6}$$

$$4(x-6) = 1x$$

$$4x - 24 = 1x$$

$$-4x \quad -4x$$

$$\frac{-24}{-3} = \frac{-3x}{-3}$$

$$\boxed{8 = x}$$

Steps: (#43 – 47)

1) Use the minus to divide rule to write the left side with one logarithm.

2) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

3) Solve for x (you will need to cross multiply to solve)

4) Check

47) $\log_2(x+6) - \log_2(3x+2) = -1$

$$\log_2 \frac{x+6}{3x+2} = -1$$

$$2^{-1} = \frac{x+6}{3x+2}$$

$$\frac{1}{2} = \frac{x+6}{3x+2}$$

$$3x+2 = 2(x+6)$$

$$3x+2 = 2x+12$$

$$-2x-2 \quad -2x-2$$

$$\boxed{x = 10}$$

1) Use the plus to times rule to write the left side with one logarithm.
2) Simplify the argument by performing the multiplication.
3) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

4) Solve for x (you will need set to equal zero and solve by factoring)

5) Check

49) $\log_3 x + \log_3 (x+6) = 3$

$\log_3 x(x+6) = 3$

$3^3 = x^2+6x$

$27 = x^2+6x$

$0 = x^2+6x-27$

$0 = (x+9)(x-3)$

$x+9=0$ $\quad x-3=0$

$x=-9$ $\quad x=3$

only $x=3$ checks

1) Use the plus to times rule to write the left side with one logarithm.
2) Simplify the argument by performing the multiplication.
3) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

4) Solve for x (you will need set to equal zero and solve by factoring)

5) Check

51) $\log_3(x+6) + \log_3(3x) = 4$

$\log_3(x+6)(3x) = 4$

$3^4 = 3x^2+6x$

$81 = 3x^2+6x$

$0 = 3x^2+6x-81$

$0 = 3(x^2+6x-27)$

$0 = 3(x+9)(x-3)$

$3 \neq 0$ No solution

$x+9=0$ $x=-9$ Does not check

$x-3=0$ $x=3$

$x=3$