

Section 6.6: Logarithmic and exponential equations

#1 - 12: Solve the exponential equation by writing each side of the equation with the same base then equating the exponents. Problems also may be solved with logarithms.

$$16 = 2^4$$

1)  $2^x = 16$

$$2^x = 2^4$$

$$\boxed{x = 4}$$

$$32 = 2^5$$

3)  $2^{x+1} = 32$

$$2^{x+1} = 2^5$$

$$\begin{array}{r} x+1 = 5 \\ -1 \quad -1 \\ \hline \end{array}$$

$$\boxed{x = 4}$$

$$5) \left(\frac{1}{2}\right)^x = 16$$

$$(2^{-1})^x = 2^4$$

$$2^{-1x} = 2^4$$

$$\frac{-1x}{-1} = \frac{4}{-1}$$

$$x = -4$$

$$7) 2^{4-x} = 64$$

$$2^{4-x} = 2^6$$

$$4 - 1x = 6$$

$$\frac{-4}{-4} \quad \frac{-4}{-4}$$

$$\frac{-1x}{-1} = \frac{2}{-1}$$

$$x = -2$$

9)  $32^x = 2$

$$(2^5)^x = 2^1$$

$$2^{5x} = 2^1$$

$$\frac{5x}{5} = \frac{1}{5}$$

$$x = \frac{1}{5}$$

11)  $16^x = 4$

$$(2^4)^x = 2^2$$

$$2^{4x} = 2^2$$

$$\frac{4x}{4} = \frac{2}{4}$$

$$x = \frac{1}{2}$$

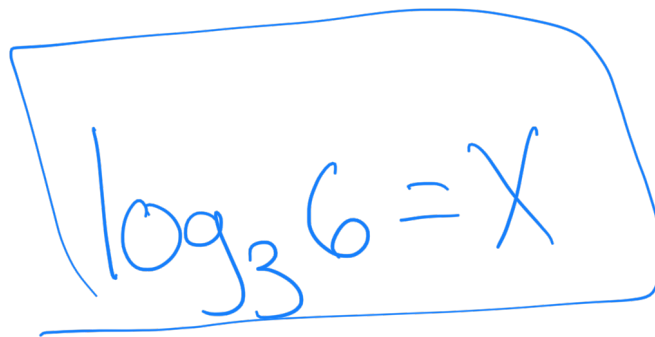
$$x = \frac{1}{2}$$

#13 - 24: Solve the exponential equations, round your answer to 2 decimals.

Steps:

- 1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)
- 2) Write in logarithmic form
- 3) Check

13)  $3^x = 6$



A handwritten equation  $\log_3 6 = x$  is enclosed in a blue hand-drawn box. The box has a slightly irregular shape with rounded corners and a horizontal line at the bottom.



Steps:

- 1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)
- 2) Write in logarithmic form (replace  $\log_e$  with  $\ln$ )
- 3) Check

15)  $e^x = 12$

$$\log_e 12 = x$$

Rewrite  $\log_e = \ln$

$$\boxed{\ln(12) = x}$$

Steps:

- 1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)
- 2) Write in logarithmic form
- 3) Check

17)  $5(10^x) = 20$   
 $\frac{5(10^x)}{5} = \frac{20}{5}$

$$10^x = 4$$

$$\log_{10} 4 = x$$

Rewrite  $\log_{10} = \log$

$$x = \log 4$$

Steps:

- 1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)
- 2) Write in logarithmic form
- 3) Check

$$19) \frac{32e^{2x}}{32} = \frac{128}{32}$$

$$e^{2x} = 4$$

$$\log_e 4 = 2x$$

$$\frac{\ln(4)}{2} = \frac{2x}{2}$$

$$x = \frac{\ln(4)}{2}$$

21)  $3^{x-1} = 5$

$$\begin{array}{ccc} & \log_3 5 = x - 1 & \\ +1 & & +1 \end{array}$$

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Steps:

1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)

2) Write in logarithmic form

3) Check

$$1 + \log_3 5 = x$$

Steps:

1) Isolate the exponential function (when needed divide by the coefficient of the exponential function, only needed for problems 17, 18, 19 and 20)

2) Write in logarithmic form

3) Check

23)  $6^{x+4} = 9$

$$\log_6 9 = x + 4$$

$$\boxed{-4 + \log_6 9 = x}$$

#25 - 51: Solve the logarithmic equations, round to 2 decimals when needed.

Steps (#25 – 38)

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

25)  $\log_3 x = 2$

$$3^2 = x$$
$$\boxed{9 = x}$$

Steps (#25 – 38)

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

27)  $\ln x = 1$

$$\log_e x = 1$$

$$e^1 = x$$

$$\boxed{x = e}$$

Steps (#25 – 38)

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

29)  $\log_x 49 = 2$  ( $x > 0$ )

$$x^2 = 49$$

$$x = \pm 7$$

only choose  $x=7$   
base must be  
positive

$$\boxed{x=7}$$



Steps (#25 – 38)

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

31)  $\log_x 64 = 3$  ( $x > 0$ )

The image shows handwritten work in blue and red ink. On the left, a red bracket groups the number '3' and the variable 'x', with '3' written above 'x'. To the right, another red bracket groups the number '3' and the number '64', with '3' written above '64'. Below these, the equation  $x = 4$  is written in blue and enclosed in a blue rounded rectangular box.

Steps (#25 – 38)

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

$$x^{1/2} = 3$$

33)  $\log_x 3 = \frac{1}{2}$  ( $x > 0$ )

$$\sqrt{x} = 3$$

$$\sqrt{x}^2 = 3^2$$

$$x = 9$$

Steps (#25 – 38)

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

$$2^3 = x - 1$$

35)  $\log_2(x-1) = 3$

$$\begin{array}{r} 8 = x - 1 \\ + 1 \quad + 1 \\ \hline 9 = x \end{array}$$

Steps (#25 – 38)

1) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

2) Solve for x

3) Check

$$2^5 = 2x$$

37)  $\log_2(2x)=5$

$$\frac{2^5}{2} = \frac{2x}{2}$$

$$16 = x$$

Steps: (#39 – 42)

1) Drop the logs and set the arguments equal to each other.

2) Solve for x.

3) Check.

39)  $\log(x+1) = \log(3x-2)$

$$\begin{array}{r} x+1 = 3x-2 \\ -x \qquad -x \\ \hline 1 = 2x-2 \\ +2 \qquad +2 \\ \hline 3 = 2x \\ \frac{3}{2} = \frac{2x}{2} \\ \boxed{\frac{3}{2} = x} \end{array}$$

Steps: (#39 – 42)

1) Drop the logs and set the arguments equal to each other.

2) Solve for x.

3) Check.

41)  $\log_2(x+3) = \log_2(3x)$

$$\begin{array}{r} x+3 = 3x \\ -x \qquad -x \\ \hline 3 = 2x \\ \frac{3}{2} = \frac{2x}{2} \\ \boxed{\frac{3}{2} = x} \end{array}$$

Steps: (#43 – 47)

1) Use the minus to divide rule to write the left side with one logarithm.

2) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base to of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

3) Solve for x (you will need to cross multiply to solve)

4) Check

43)  $\log_2 x - \log_2 (x+6) = -2$

$$\log_2 \frac{x}{x+6} = -2$$

$$2^{-2} = \frac{x}{x+6}$$

~~$$\frac{1}{2^2} = \frac{x}{x+6}$$~~

$$\frac{1}{4} = \frac{x}{x+6}$$

$$\begin{array}{r} x+6 = 4x \\ -x \quad \quad -x \\ \hline 6 = 3x \end{array} \quad \sqrt{2=x}$$

Steps: (#43 – 47)

1) Use the minus to divide rule to write the left side with one logarithm.

2) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

3) Solve for x (you will need to cross multiply to solve)

4) Check

$$\log_2 \frac{x}{x-6} = 2$$

45)  $\log_2 x - \log_2 (x - 6) = 2$

$$2^2 = \frac{x}{x-6}$$

$$4 = \frac{x}{x-6}$$

$$\frac{4}{1} = \frac{x}{x-6}$$

$$\begin{array}{r} 4(x-6) = 1x \\ 4x - 24 = 1x \\ \underline{-4x} \qquad \underline{-4x} \end{array}$$

$$\frac{-24}{-3} = \frac{-3x}{-3}$$

$$\boxed{8 = x}$$



Steps: (#43 – 47)

1) Use the minus to divide rule to write the left side with one logarithm.

2) Rewrite the problem in exponential form.

- Scratch out the log and create an exponential function.
- The base of the logarithm will be the base of the exponential function.
- Switch: make the number to the right of the equal sign an exponent
- place the argument to the right of the equal sign.

3) Solve for x (you will need to cross multiply to solve)

4) Check

$$\log_2 \frac{x+6}{3x+2} = -1$$

47)  $\log_2(x+6) - \log_2(3x+2) = -1$

$$2^{-1} = \frac{x+6}{3x+2}$$

$$\frac{1}{2} = \frac{x+6}{3x+2}$$

$$3x+2 = 2(x+6)$$

$$\begin{array}{r} 3x+2 = 2x+12 \\ -2x-2 \quad -2x-2 \\ \hline \end{array}$$

$$\boxed{x = 10}$$

- 1) Use the plus to times rule to write the left side with one logarithm.
- 2) Simplify the argument by performing the multiplication.
- 3) Rewrite the problem in exponential form.
  - Scratch out the log and create an exponential function.
  - The base of the logarithm will be the base of the exponential function.
  - Switch: make the number to the right of the equal sign an exponent
  - place the argument to the right of the equal sign.
- 4) Solve for x (you will need set to equal zero and solve by factoring)
- 5) Check

$$\log_3 x(x+6) = 3$$

$$3^3 = x^2 + 6x$$

$$27 = x^2 + 6x$$

$$0 = x^2 + 6x - 27$$

$$0 = (x+9)(x-3)$$

$$x+9=0$$

$$x-3=0$$

$$x=-9$$

$$x=3$$

49)  $\log_3 x + \log_3 (x+6) = 3$

only  $x=3$  checks

- 1) Use the plus to times rule to write the left side with one logarithm.
- 2) Simplify the argument by performing the multiplication.
- 3) Rewrite the problem in exponential form.
  - Scratch out the log and create an exponential function.
  - The base of the logarithm will be the base of the exponential function.
  - Switch: make the number to the right of the equal sign an exponent
  - place the argument to the right of the equal sign.

4) Solve for x (you will need set to equal zero and solve by factoring)

5) Check

$$\log_3(x+6)(3x) = 4$$

51)  $\log_3(x+6) + \log_3(3x) = 4$

$$x=3$$

$$3^4 = 3x^2 + 6x$$

$$81 = 3x^2 + 6x$$

$$0 = 3x^2 + 6x - 81$$

$$0 = 3(x^2 + 6x - 27)$$

$$0 = 3(x+9)(x-3)$$

$3=0$   
 No x  
 so no  
 solution

$x+9=0$        $x-3=0$   
 $x=-9$            $x=3$   
 Does  
 not  
 check